

# Chemical amplification at the wave head of a finite amplitude gasdynamic disturbance

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Consider a background state which consists of a spatially uniform chemically reactive mixture in a general state of disequilibrium. The analytical method of characteristics is used to show that a plane finite amplitude disturbance propagates through this system at the frozen sound speed and, if the degree of disequilibrium is sufficient, is amplified by the chemical reaction. Some comments are made about the time to shock-wave formation and its relation to the homogeneous explosion ignition time, and also about expansion waves, which are found to have a tendency towards fixed-strength 'quenching waves', their strength being proportional to the extent of the ambient disequilibrium.

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## 1. Introduction

When a gasdynamic disturbance propagates through a chemically reacting gas mixture that is in a state of chemical equilibrium, the perturbed reaction acts to damp out the disturbance. The behaviour of infinitesimal amplitude (acoustic) disturbances and of disturbances of finite amplitude is quite well understood, although some interesting problems remain to be solved. Much less interest has been shown in the propagation of waves through systems that are not in chemical equilibrium, exceptions being the work of Srinivasan & Vincenti (1975) and the writer (1973, 1974), all of whom have been concerned with acoustic phenomena.

The principal result of these acoustic theories is that, under the proper conditions, the basic disequilibrium in the background state can lead to disturbance amplification rather than decay. Such a possibility is evidently important in a whole variety of situations where chemical reactions take place, and not the least significant of these concerns explosive gas mixtures. The acoustic theories are generally limited to prediction of initial amplification, and cannot follow the history of a developing, finite amplitude, wave.

If attention is confined to the head of a wave it is possible to make exact predictions of the true nonlinear progress of gas-property gradients at the wave front and in this way to calculate such things as the time taken to form a shock wave. The present paper considers a simple model of a reaction with Arrhenius kinetics which is also reversible, since the extent of the background disequilibrium is vital, and derives the time history of the velocity gradient at the plane wave head.

Using simplifying assumptions to make the exact result more accessible, this exploratory investigation shows that the time to shock formation can be comparable

with the homogeneous-explosion ignition time, and that expansion waves may have a tendency to form into fixed-strength disturbances ('quenching waves'?) with the strength depending upon the basic disequilibrium state. Apart from the direct application to homogeneous explosions, identification of chemical reaction as an amplifier of gasdynamic disturbances may also make the results important in combustion ignition and extinction studies and in an understanding of combustion-generated turbulence and noise.

## 2. Equations in characteristic form

The equations governing the one-dimensional unsteady motion of a reacting gas mixture are (Clarke & McChesney 1976, chap. 1, especially §1.18).

$$\rho_t + u\rho_x + \rho u_x = 0, \quad (1)$$

$$\rho u_t + \rho u u_x + p_x = 0, \quad (2)$$

$$p_t + u p_x + \rho a_f^2 u_x = \rho(\gamma - 1) Q W n \mathcal{R}, \quad (3)$$

$$c_t + u c_x = - W n \mathcal{R}. \quad (4)$$

Transport effects have been neglected and the symbols appearing in (1)–(4) are as follows:  $\rho$  is the density;  $p$  is the pressure;  $u$  is the gas velocity;  $c$  is the mass fraction of the reactant species  $A$ , which takes part in the simple reversible reaction



( $B$  is the product species, which can dissociate to produce  $n$  molecules of  $A$ );  $W$  is the molecular weight of  $A$  (strictly speaking  $W$  will vary with  $c$ : suppression of this dependence simplifies the subsequent algebra and does not radically affect the final results);  $\gamma$  is the not necessarily constant ratio of specific heats of the gas mixture under chemically frozen conditions;  $a_f$  is the frozen sound speed, which, if the constituents of the gas mixture are assumed to be thermally perfect, is given by

$$a_f^2 = \gamma p / \rho; \quad (6)$$

$Q$  is the energy of formation per unit mass of  $A$ , and  $\mathcal{R}$  is the rate of progress of reaction (5), namely

$$\mathcal{R} = \tau^{-1} \{ c^n - (1 - c) \delta \}. \quad (7)$$

The quantities  $\tau$  and  $\delta$  are the forward-reaction time,

$$\tau^{-1} = F(p/\rho) \exp \{ - E_A \rho / p \}, \quad (8)$$

and the equilibrium constant,

$$\delta = z(p/\rho) \exp \{ - n Q \rho / p \}, \quad (9)$$

respectively. The pre-exponential factor  $F$  in (8) and the coefficient  $z$  in (9) depend upon  $p/\rho$ , but much more weakly than the exponential terms that are explicitly displayed in those equations, especially since both  $Q$  and  $E_A$ , the reaction's activation energy, are usually large relative to  $p/\rho$ .  $F$  and  $z$  will henceforth be treated as constants; it does not fundamentally affect the results to do this, and their proper variations with  $p/\rho$  can be included at the simple expense of more algebra.

With the auxiliary relations (6)–(9), equations (1)–(4) constitute four relations for the four unknowns,  $p$ ,  $\rho$ ,  $u$  and  $c$ . Equations (2) and (3) combine to give

$$p_\alpha + \rho a_f u_\alpha - \rho(\gamma - 1) Q W n \mathcal{R} t_\alpha = 0, \tag{10}$$

$$p_\beta - \rho a_f u_\beta - \rho(\gamma - 1) Q W n \mathcal{R} t_\beta = 0, \tag{11}$$

where  $\alpha$  and  $\beta$  are two characteristic parameters defined such that

$$(dx/dt)_{\beta \text{ fixed}} = x_\alpha / t_\alpha = u + a_f, \tag{12}$$

$$(dx/dt)_{\alpha \text{ fixed}} = x_\beta / t_\beta = u - a_f. \tag{13}$$

Transformation from the space–time  $(x, t)$  plane to the plane of the characteristic parameters  $(\alpha, \beta)$  will be one-to-one if the Jacobian

$$J = x_\alpha t_\beta - x_\beta t_\alpha = 2a_f t_\alpha t_\beta \tag{14}$$

does not vanish anywhere. The following results are noted for future use:

$$f_t + u f_x = a_f J^{-1} \{ f_\alpha t_\beta + f_\beta t_\alpha \}, \tag{15}$$

$$f_x = J^{-1} \{ f_\alpha t_\beta - f_\beta t_\alpha \}, \tag{16}$$

where  $f$  can be either  $p$ ,  $\rho$ ,  $u$  or  $c$ .

### 3. The background state

The unperturbed field ahead of the wave whose behaviour is to be investigated is assumed to be spatially uniform. All  $x$  derivatives therefore vanish and (1)–(4) give

$$\rho_{0t} = 0, \quad \text{so that} \quad \rho_0 = \text{constant}, \tag{17}$$

$$\rho_0 u_{0t} = 0, \quad \text{so that} \quad u_0 = 0, \tag{18}$$

$$p_{0t} = \rho_0(\gamma - 1) Q W n \mathcal{R}_0, \tag{19}$$

$$c_{0t} = - W n \mathcal{R}_0. \tag{20}$$

A subscript 0 indicates a value in the background field, and its absence from  $\gamma$  in (19) implies that  $\gamma$  will henceforth be treated as a constant.

Evidently the background state can be visualized in terms of a fixed vessel uniformly filled [equation (17)] with the gas mixture, which is at rest [equation (18)]; the pressure  $p_0$  changes [equation (19)] as the reactant-species concentration  $c_0$  varies [equation (20)], namely in proportion to the ambient reaction rate  $\mathcal{R}_0$ . The latter would be zero if the chemical time  $\tau$  happened to be infinite or, more practically, if the background state were one of chemical equilibrium.

Perturbations of the background state will be assumed to propagate through the mixture behind the wave front  $\beta = 0$  (see figure 1). Continuity of the variables  $p$ ,  $\rho$ ,  $u$  and  $c$  at  $\beta = 0$  is essential but discontinuities in their derivatives are permitted. Any derivative with respect to  $\alpha$  must be continuous; discontinuities can appear only in the  $\beta$  derivatives. The associated discontinuities in the  $x$  derivatives follow from (16).

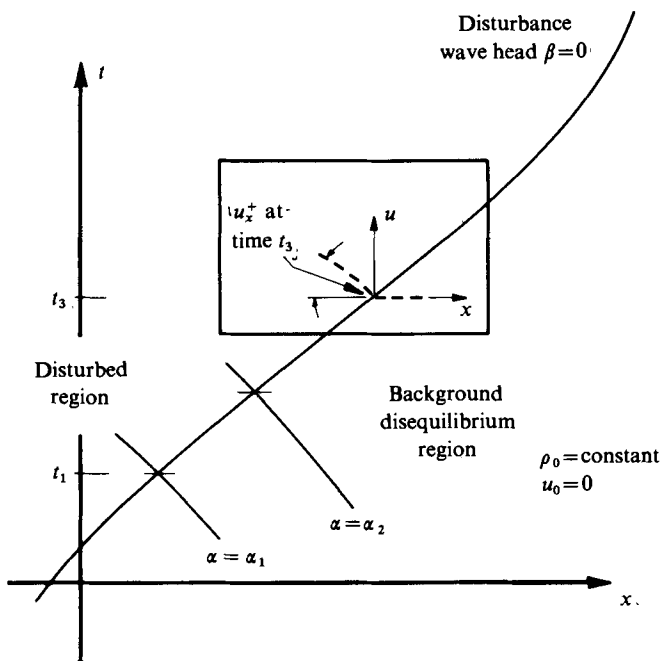


FIGURE 1. The  $x, t$  plane, showing the disturbance wave head  $\beta = 0$  and two typical constant- $\alpha$  lines. The inset sketch shows the type of discontinuity in  $u_x$  at  $\beta = 0$  which heralds a compression wave.

**4. Behaviour at the wave front**

Differentiation of (10) with respect to  $\beta$  and of (11) with respect to  $\alpha$  followed by subtraction and evaluation at  $\beta = 0 +$  gives

$$2\rho_0 a_{f0} u_{\alpha\beta}^+ + (\rho_0 a_{f0})_{\alpha} u_{\beta}^+ - (\gamma - 1) Q W n \{ \rho_{\beta}^+ t_{0\alpha} \mathcal{R}_0 + \rho_0 (t_{0\alpha} \mathcal{R}_{\beta}^+ - \mathcal{R}_{0\alpha} t_{\beta}^+) \} = 0. \tag{21}$$

Quantities with a subscript 0 are suitably continuous in the neighbourhood of  $\beta = 0$  while those with a superscript + may be discontinuous at the wave head, so that their value just behind the front is required in (21).

Equations (1), (15) and (16) show that

$$a_f \{ \rho_{\alpha} t_{\beta} + \rho_{\beta} t_{\alpha} \} + \rho \{ u_{\alpha} t_{\beta} - u_{\beta} t_{\alpha} \} = 0, \tag{22}$$

whence it follows from (17) and (18) that

$$a_{f0} \rho_{\beta}^+ = \rho_0 u_{\beta}^+. \tag{23}$$

From the forms of  $\mathcal{R}, \tau$  and  $\delta$  given in (7)–(9) it transpires that

$$t_{0\alpha} \mathcal{R}_{\beta}^+ - \mathcal{R}_{0\alpha} t_{\beta}^+ = - \left( \frac{E_A \rho_0}{p_0} \mathcal{R}_0 - (1 - c_0) \frac{\delta_0}{\tau_0} Q \frac{\rho_0}{p_0} \right) \left( \frac{t_{0\alpha} \rho_{\beta}^+}{\rho_0} - \frac{1}{p_0} [t_{0\alpha} p_{\beta}^+ - p_{0\alpha} t_{\beta}^+] \right) + \frac{1}{\tau_0} (n c_0^{n-1} + \delta_0) (t_{0\alpha} c_{\beta}^+ - c_{0\alpha} t_{\beta}^+). \tag{24}$$

Equations (2), (15) and (16) give

$$\rho a_f(u_\alpha t_\beta + u_\beta t_\alpha) + (p_\alpha t_\beta - p_\beta t_\alpha) = 0, \tag{25}$$

so that

$$t_{0\alpha} p_\beta^+ - p_{0\alpha} t_\beta^+ = \rho_0 a_{f0} t_{0\alpha} u_\beta^+. \tag{26}$$

From the essential continuity of  $\mathcal{R}$ , (4) shows that  $c_t + u c_x$  must be continuous at  $\beta = 0$ ; (15) therefore implies that

$$\frac{c_{0\alpha}}{2a_{f0} t_{0\alpha}} + \frac{c_\beta^+}{2a_{f0} t_\beta^+} = \frac{c_{0\alpha}}{2a_{f0} t_{0\alpha}} + \frac{c_{0\beta}}{2a_{f0} t_{0\beta}},$$

and so

$$\begin{aligned} t_{0\alpha} c_\beta^+ - c_{0\alpha} t_\beta^+ &= t_{0\alpha} c_{0\beta} t_\beta^+ / t_{0\beta} - c_{0\alpha} t_\beta^+ \\ &= [c_{0\beta} t_{0\alpha} - c_{0\alpha} t_{0\beta}] t_\beta^+ / t_{0\beta} \\ &= 0. \end{aligned} \tag{27}$$

The final result follows because the quantity in square brackets is  $c_{0x}$ , and this is zero by hypothesis.

Combining (21) with (23), (24), (26) and (27) gives the following equation for the variation of  $u_\beta^+$  with  $\alpha$ :

$$\frac{\partial}{\partial \alpha} \ln \{(\rho_0 a_{f0})^{\frac{1}{2}} u_\beta^+\} = \frac{1}{2}(\gamma - 1) Q W n \left\{ \frac{(\gamma - 1) E_A \rho_0}{p_0 a_{f0}^2} \left( \mathcal{R}_0 - (1 - c_0) \frac{\delta_0}{\tau_0} E_A \right) + \frac{\mathcal{R}_0}{a_{f0}^2} \right\} t_{0\alpha}. \tag{28}$$

Integrating (28) between  $\alpha_i$  (where  $u_\beta^+ = u_{\beta i}^+$  and  $t = t_i$ ) and  $\alpha$  gives [note (6)]

$$u_\beta^+ = u_{\beta i}^+ \left( \frac{\rho_{0i} a_{f0i}}{\rho_0 a_{f0}} \right)^{\frac{1}{2}} \exp \left\{ \frac{1}{2} \frac{(\gamma - 1)^2}{\gamma} Q E_A W n \int_{t_i}^t \frac{\rho_0^2}{p_0^2} \left[ \mathcal{R}_0 - (1 - c_0) \frac{\delta_0}{\tau_0} \frac{Q}{E_A} \right] dt \right\}. \tag{29}$$

Since  $E_A \rho_0 / p_0 \gg 1$  for many practical situations (values of 50–100 are typical for normal temperatures and pressures), the term  $\mathcal{R}_0 / a_{f0}^2$  in the brace brackets in (28) has been omitted in presenting the result (29).

In order to discover how  $u_x$ , say, is changing at  $\beta = 0+$  it is necessary to find  $t_\beta^+$ , since (14), (16) and (18) give

$$u_x^+ = -u_\beta^+ / 2a_{f0} t_\beta^+. \tag{30}$$

Equations (12) and (13) combine to give

$$2a_f t_{\alpha\beta} + (u_\beta + a_{f\beta}) t_\alpha - (u_\alpha - a_{f\alpha}) t_\beta = 0, \tag{31}$$

so that at  $\beta = 0+$   $2a_{f0} (t_\beta^+)_\alpha + u_\beta^+ t_{0\alpha} + \{a_{f\beta}^+ t_{0\alpha} + a_{f0\alpha} t_\beta^+\} = 0. \tag{32}$

But

$$a_{f\beta} t_\alpha + a_{f\alpha} t_\beta = \left( \frac{\partial a_f}{\partial p} \right)_{\rho, c} (p_\beta t_\alpha + p_\alpha t_\beta) + \left( \frac{\partial a_f}{\partial \rho} \right)_{p, c} (\rho_\beta t_\alpha + \rho_\alpha t_\beta) \tag{33}$$

since (6) shows that  $(\partial a_f / \partial c)_{p, \rho}$  vanishes when  $\gamma$  is constant. The  $\rho$  derivative combination in (33) can be eliminated with the aid of (22); the  $p$  derivatives can be written in more convenient terms by combining (10) and (11) to give

$$p_\alpha t_\beta + p_\beta t_\alpha = \rho a_f (u_\beta t_\alpha - u_\alpha t_\beta) + 2\rho(\gamma - 1) Q W n \mathcal{R} t_\alpha t_\beta. \tag{34}$$

Then (32) with (33) and (22) becomes

$$(t_\beta^+)_\alpha + \frac{1}{2}(\gamma - 1) Q W n \frac{\mathcal{R}_0}{(p_0 / \rho_0)} t_{0\alpha} t_\beta^+ + \frac{1}{4} \frac{(\gamma + 1)}{a_{f0}} u_\beta^+ t_{0\alpha} = 0, \tag{35}$$

which, together with the result (29), can be integrated to find the variation of  $t_{\beta}^+$  with time at the wave head.

Defining 
$$\Lambda_1 = \frac{1}{2} \frac{(\gamma - 1)^2 Q E_A W n}{\gamma (p_0/\rho_0)^2} \left[ \mathcal{R}_0 - (1 - c_0) \frac{\delta_0}{\tau_0} \frac{Q}{E_A} \right], \tag{36}$$

$$\Lambda_2 = \frac{1}{2} (\gamma - 1) \frac{Q}{(p_0/\rho_0)} W n \mathcal{R}_0, \tag{37}$$

equation (35) gives

$$t_{\beta}^+ = t_{\beta i}^+ \exp \left\{ - \int_{t_i}^t \Lambda_2(\tilde{t}) d\tilde{t} \right\} - u_{\beta i}^+ \exp \left\{ - \int_{t_i}^t \Lambda_2(\tilde{t}) d\tilde{t} \right\} \int_{t_i}^t \frac{1}{4} \frac{(\gamma + 1)}{a_{f_0}} \left( \frac{\rho_{0i} a_{f_{0i}}}{\rho_0 a_{f_0}} \right)^{\frac{1}{2}} \exp \left\{ \int_{t_i}^{\tilde{t}} \Lambda(\hat{t}) d\hat{t} \right\} d\tilde{t}, \tag{38}$$

where 
$$\Lambda = \Lambda_1 + \Lambda_2 \tag{39}$$

and  $u_{\beta i}^+$  is the value of  $u_{\beta}^+$  when  $t = t_i$ .

Combination of (30), (38) and (29), which yields

$$u_{\beta}^+ = u_{\beta i}^+ \left( \frac{\rho_{0i} a_{f_{0i}}}{\rho_0 a_{f_0}} \right)^{\frac{1}{2}} \exp \left\{ \int_{t_i}^t \Lambda_1(\tilde{t}) d\tilde{t} \right\} \tag{40}$$

in view of (36), now enables one to evaluate  $u_x^+$ . Noting that (30) makes

$$u_{xi}^+ = -u_{\beta i}^+ / 2a_{f_{0i}} t_{\beta i}^+, \tag{41}$$

the result is

$$\frac{u_x^+}{u_{xi}^+} = \left( \frac{\rho_{0i} a_{f_{0i}}^3}{\rho_0 a_{f_0}^3} \right)^{\frac{1}{2}} \frac{\exp \left\{ \int_{t_i}^t \Lambda(\tilde{t}) d\tilde{t} \right\}}{\left[ 1 + \frac{1}{2} (\gamma + 1) u_{xi}^+ \int_{t_i}^t \left( \frac{\rho_{0i} a_{f_{0i}}^3}{\rho_0 a_{f_0}^3} \right)^{\frac{1}{2}} \exp \left\{ \int_{t_i}^{\tilde{t}} \Lambda(\hat{t}) d\hat{t} \right\} d\tilde{t} \right]}. \tag{42}$$

The significance of this general result is not immediately apparent since the dependence of  $u_x^+$  on the background conditions is obscured by the integrals of  $\Lambda(t)$ , etc., and by the quite complex dependence of  $\Lambda$  on this state via (36), (37) and (39). In view of the approximations that have already been made, especially the one between (28) and (29), some simplification is afforded by neglecting  $\Lambda_2$  relative to  $\Lambda_1$ . In any event it is clear from (42) that the temporal behaviour of the velocity gradient at the wave head will depend critically on the sign of  $\Lambda$ .

### 5. Discussion

If  $\mathcal{R}_0$  is zero, so that the background state is one of uniform equilibrium, (36) and (37) show that

$$\Lambda = \Lambda_1 = -\frac{1}{2} \frac{(\gamma - 1)^2 Q^2 W n}{\gamma (p_0/\rho_0^2)} (1 - c_0) \frac{\delta_0}{\tau_0} < 0. \tag{43}$$

Then (42) reduces to the well-known result for propagation through equilibrium atmospheres that has been derived by several previous investigators (a short account is given by Clarke & McChesney 1976, § 2.4).

If one considers only a short interval of time, so that  $\Lambda$  does not change appreciably

between  $t_i$  and  $t$ , it is evident that (42) can be written in the approximate form

$$\frac{u_x^+}{u_{xi}^+} \simeq \frac{e^{\bar{\Lambda}t}}{1 + \frac{1}{2}\bar{\Lambda}^{-1}(\gamma + 1)u_{xi}^+(e^{\bar{\Lambda}t} - 1)}, \quad (44)$$

where  $\bar{\Lambda}$  indicates a suitable mean value over the interval  $t_i$  to  $t$ , and  $t_i$  has been set equal to zero for convenience; it easily follows that

$$\frac{d}{dt} \left( \frac{u_x^+}{u_{xi}^+} \right) \simeq \frac{\{\bar{\Lambda} - \frac{1}{2}(\gamma + 1)u_{xi}^+\} e^{\bar{\Lambda}t}}{[1 + \frac{1}{2}\bar{\Lambda}^{-1}(\gamma + 1)u_{xi}^+(e^{\bar{\Lambda}t} - 1)]^2}. \quad (45)$$

Equation (45) shows that, if  $u_{xi}^+ < 0$  and  $\bar{\Lambda} > 0$ ,  $u_x^+$  will begin to grow even more negative as time proceeds, and the strength of the compression wave head (see figure 1) will therefore grow with time. Of course this happens to a compression wave in an inert gas anyway, and the corresponding result for the chemically inert system is

$$\left. \frac{d}{dt} \left( \frac{u_x^+}{u_{xi}^+} \right) \right|_{\text{inert}} = \frac{-\frac{1}{2}(\gamma + 1)u_{xi}^+}{[1 + \frac{1}{2}(\gamma + 1)u_{xi}^+t]^2}. \quad (46)$$

Comparison of (45) and (46) shows, as one might expect, that the exothermic reaction with its Arrhenius rate dependence causes the compression wave to steepen more swiftly than it does in an inert atmosphere. Indeed, if (44) remains valid over the requisite period, it shows that  $|u_x^+| \rightarrow \infty$  when  $t \rightarrow t_s$ , where

$$t_s = \frac{1}{\bar{\Lambda}} \ln \left\{ 1 + \frac{2\bar{\Lambda}}{(\gamma + 1)|u_{xi}^+|} \right\}. \quad (47)$$

[This corresponds to the vanishing of the Jacobian since  $t_\beta = 0$ ; see (14).]

From time  $t_s$  onwards the compression wave must be treated as a shock wave; in the present exothermic situation its subsequent behaviour could become very interesting indeed. For example, one may inquire into the magnitude of  $t_s$  relative to the time taken for the background homogeneous mixture to ignite, since it will be interesting to know whether a shock forms before, during or after this spontaneous event. It is during this thermal run-away process that  $\Lambda$  will reach its maximum possible values.

For an exact result it is of course necessary to evaluate all of the component terms in  $\Lambda(r)$  by solving the problem for the ambient atmosphere described by (17)–(20). This is quite a challenging problem in its own right in the theory of explosions, and it has been considered extensively in the past, using a number of different methods and approximations. For present purposes it is quite adequate to consider the simple ‘small depletion’ model of the explosion-initiation, or ignition, history. The reverse reaction in (7) is ignored and it is assumed that  $c$ , or rather  $c_0$  in this case, is a constant for all practical purposes; say,  $c_0 \simeq c_{0i}$ . Then (19) reads

$$p_{0t} \simeq \rho_0(\gamma - 1)QWnFc_{0i}^n \exp(-E_A\rho_0/p_0) \quad (48)$$

and, since  $p_0$  is the only variable in (48), it can be integrated directly. Even then the result is not especially informative, but it can be made so by adopting the approximation  $E_A\rho_0/p_0 \gg 1$ . It is found that the time required to raise  $p_0$  from  $p_{0i}$  to the value  $mp_{0i}$  ( $m > 1$ ) is given by  $t_m$ , where

$$\rho_0(\gamma - 1)QWnFc_{0i}^n t_m \simeq \exp(E_A\rho_0/p_{0i}) \frac{p_{0i}^2}{\rho_0 E_A} \left\{ 1 - m^2 \exp \left[ E_A\rho_0 \left( \frac{1}{mp_{0i}} - \frac{1}{p_{0i}} \right) \right] \right\}. \quad (49)$$

For any substantial value of  $m$ , such as 2, the last term in (49) is negligible and  $t_m$  gives a value for the ignition time  $t_{\text{ign}}$ , namely

$$t_{\text{ign}} \simeq \frac{(p_{0i}/\rho_0)^2 \exp(E_A \rho_0/p_{0i})}{(\gamma-1) Q E_A W n F c_{0i}^2} \simeq \frac{\gamma-1}{2\gamma \Lambda(t_i)}. \quad (50)$$

From this preliminary, exploratory analysis it is not possible to be more precise but  $\Lambda(t_i)$ , the initial value of  $\Lambda$ , and the mean value  $\bar{\Lambda}$  of  $\Lambda$  are evidently comparable for any times  $t \lesssim t_{\text{ign}}$ . In these circumstances a shock can evidently form prior to spontaneous ignition, or indeed at about the same time as this process, if  $|u_{xi}^+|$  is large enough. The extremely rapid thermal run-away phenomenon that follows the (relatively) slow self-heating process in the interval  $0 < t \lesssim t_{\text{ign}}$  is synonymous with very considerable rises in the positive value of  $\Lambda$  before it sinks back to its negative equilibrium value [given in (43)], and all of this suggests that shocks are quite likely to form from some initial compressive disturbance during the interval of spontaneous ignition.

In so far as (44) is a reasonable representation of the exact solution (42) it is also interesting to note that when  $u_{xi}^+ > 0$  and  $\bar{\Lambda} > 0$

$$u_x^+ \rightarrow 2\bar{\Lambda}/(\gamma+1) \quad \text{as} \quad \bar{\Lambda}t \rightarrow \infty. \quad (51)$$

Therefore an expansion wave head tends towards a strength which is independent of its initial value, and reflects instead the degree of background disequilibrium, as exemplified by  $\bar{\Lambda}$ . Of course the condition  $\bar{\Lambda}t \rightarrow \infty$  means that this asymptotic expansion wave state may be reached only after many ignition-time intervals; since  $\Lambda$  will become negative in the last stages of a homogeneous explosion, the approximation (44) cannot be valid up to these times, but the *tendency* towards a reaction-induced 'quenching wave' of some fixed form is nonetheless important.

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